**1.4.1**. Factor these matrices into:

**Sol**.

**1.4.2**. If,,is the first row of a rank-1 matrixand,,is the first column, find a formula for. Good to check when , , . When will your formula break down? Then rank 1 is impossible or not unique.

**Sol**. Check. has .

The formula breaks down when. Thenis not rank 1, oris rank 1 but not unique.

**1.4.3**. What lower triangular matrixputsinto upper triangular form? Multiply by to factorinto:

**Sol**.

**1.4.4**. This problem shows how the one-step inverses multiply to give. You see this best whenis already lower triangular with 1's on the diagonal. Then: Multiplybyand then.

(a) Multiplyto find the single matrixthat produces.

(b) Multiplyto find the matrix.

The multipliersare mixed up inbut they are perfect in.

**Sol**. (a)

(b)

**1.4.5**. When zero appears in a pivot position,is not possible! (We are requiring nonzero pivots in.) Show directly why these equations are both impossible:

These matrices need a row exchange by a permutation matrix.

**Sol**. whereis impossible.

is impossible.

**1.4.6**. Which numberleads to zero in the second pivot position? A row exchange is needed andwill not be possible.

Whichproduces zero in the third pivot position? Then a row exchange can't help and elimination fails.

**Sol**.

**1.4.7**. (Recommended) Computeandfor this symmetric matrix: . Find four conditions onto getwith four nonzero pivots.

**Sol**. where

**1.4.8**. Tridiagonal matrices have zero entries except on the main diagonal and the two adjacent diagonals. Factor these into. Symmetry further produces: and

**Sol**.

**1.4.9**. Easy but important. Ifhas pivotswith no row exchanges, what are the pivots for the upper leftbysubmatrix(without rowand column)?

**Sol**.

**1.4.10**. Which invertible matrices allow(elimination without row exchanges)? Good question! Look at each of the square upper left submatrices,,,. All upper left submatricesmust be invertible: sizes 1 by 1, 2 by 2, ... ,by. Explain that answer: factors into \_\_\_\_\_ because.

**Sol**.

**1.4.11**. In some data science applications, the first pivot is the largest numberin. Then rowbecomes the first pivot row. Column is the first pivot column. Divide that column bysohas 1 in row. Then remove thatfrom. This example findsas the first pivot (). Dividing by 4 gives:

.

For this, bothandinvolve permutations.exchanges the rows to give.exchanges the columns to give an upper triangular. Then. Permuted in advance. Question for: Apply complete pivoting to produce .

**Sol**.

**1.4.12**. If the short wide matrix has , how does elimination show that there are nonzero solutions to ? What do we know about the dimension of that “nullspace of ” containing all solution vectors ? The nullspace dimension is at least \_\_\_\_\_ .

Suggestion: First create a specific 2 by 3 matrix and ask those questions about .

**Sol**. The Row Reduced Echelon Form of gives pivot elements, where .

The number of the left columns is the number of free variables, namely the dimension of nullspace of A.

And , so the dimension is at least .